The Kelvin-Helmholtz Instability in Space

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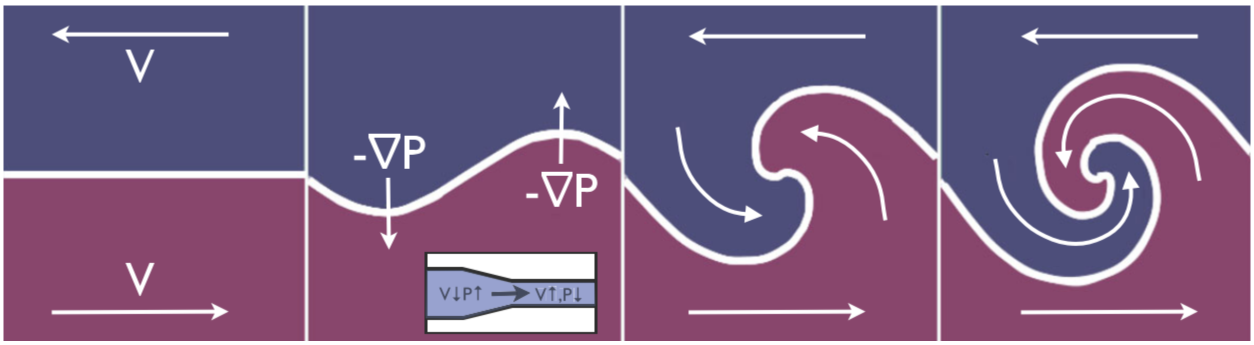


Fig. 1.—: The Kelvin-Helmholtz Instability growing on a shear boundary, from Johnson *et al.* (2014)

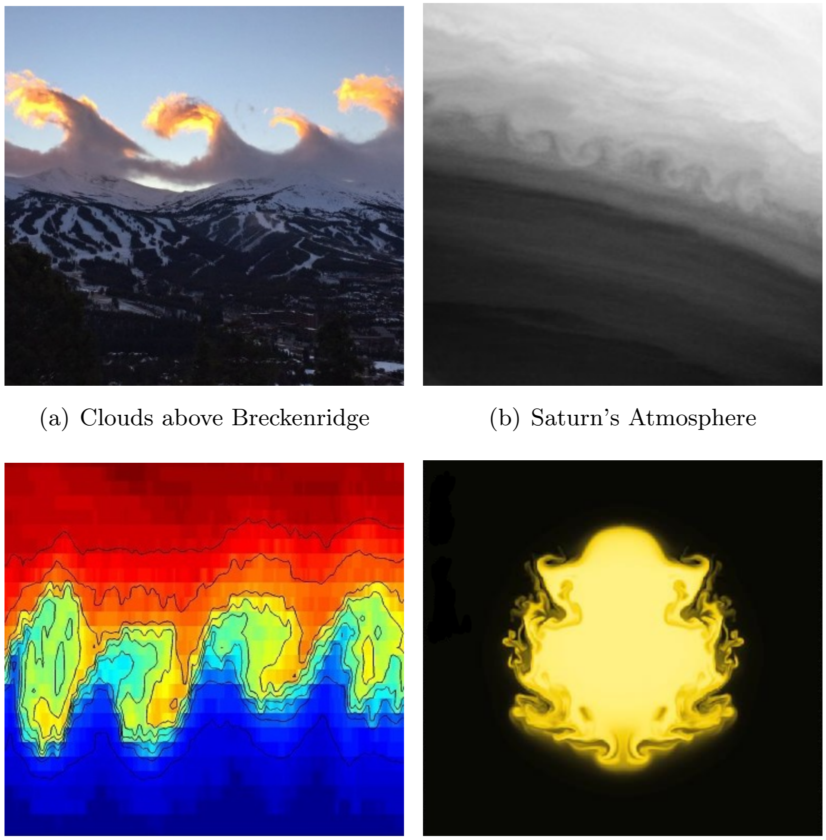
1. Introduction

The Kelvin-Helmholtz Instability (KHI) was first characterized in the later part of the 19th century by Lord Kelvin and Hermann von Helmholtz. Since then, much work has been done to study the conditions under which a sheared flow is unstable in this way, and what effect it has on the flow. The KHI is an important process that occurs in the Earth's Magnetosphere (MS). It plays a large role in the transport of mass, moment, and energy across magnetospheric boundaries, as it can modulate the development of turbulent boundary layers (Johnson *et al.* 2014). This instability was first documented by Helmholtz (1868) and Kelvin (1871), and can be characterized by the formation of vortices when there is a shearing flow. Some common examples can be seen in Figure 2. In this work, the cause of the instability is explained. Then, the role of this instability in several space physics applications is explored.

2. The Instability

2.1. Description

The KH Instability occurs when there are two adjacent fluids moving with a velocity shear between them. See Figure 5. It occurs at a tangential discontinuity (TD), which is the interface where an abrupt change in the flow velocity, density, temperature, and magnetic field is observed. (Mishin & Tomozov 2016) To understand why it happens, one must consider the Bernoulli principle. When there is a perturbation of the boundary between the two fluids, it causes a constriction of one of the fluids. This leads to increased velocity and reduced pressure. For the other fluid, the boundary is expanding, and there is reduced flow and increased pressure. Because of this, a deformation of the boundary leads to a pressure gradient which creates a pressure force in the same direction as the deformation. Thus, a small perturbation in the boundary between the two fluids will grow. As fluid from one side moves to the other side of the



(a) Clouds above Breckenridge (b) Saturn's Atmosphere

(c) Deep Ocean Currents on Earth (d) Magnetic flux tubes on the Sun

Fig. 2.—: The KH Instability occurs all over the place, at many different scales†

boundary, it is then accelerated by the surrounding fluid, and it begins to roll up. This is what causes the vortices that characterize this instability. (Johnson *et al.* 2014)

2.2. Analysis

A standard method for examining an instability is to find the dispersion relation of the system. This is an equation that relates the frequency and wavenumber of any waves that occur in a particular fluid system. To accomplish this, the Navier-Stokes Equations, as well as any other necessary equations like the MHD equations, are linearized and put in matrix form. Next, the spectral ansatz is applied, where for each variable X, one assumes the form

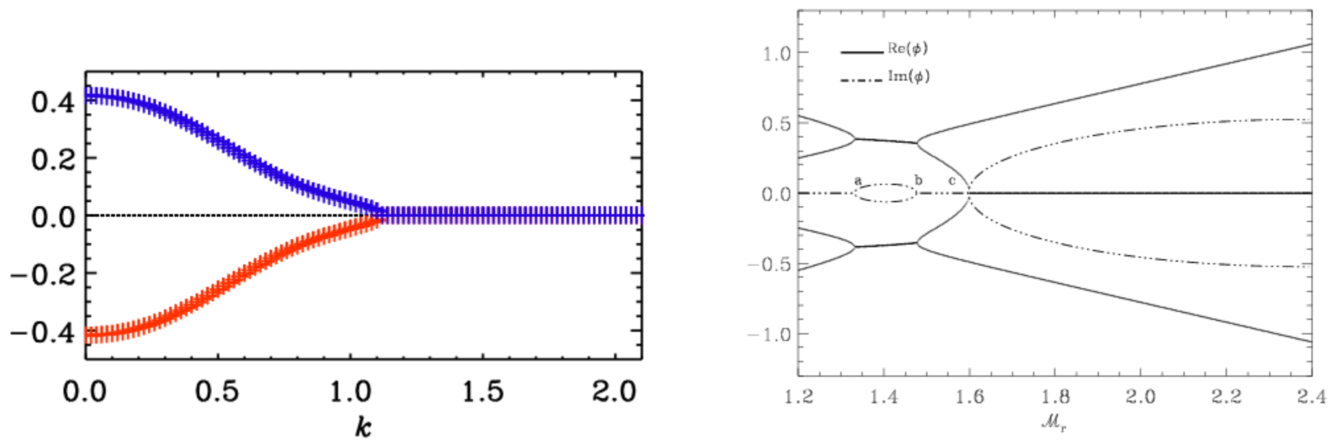
$$X = X_0 * e^{i(kz-\omega t)}, \quad \frac{dX}{dt} = -i\omega X, \quad \frac{dX}{dz} = ikX \tag{1}$$

Thus, ω and k are introduced to the equations. Then the determinant of that matrix is found and set equal to zero, and one can then solve for ω. For a slab geometry and incompressible plasma, the dispersion relation works out to be:

$$\omega = \frac{\mathbf{k}\cdot(\rho_1\mathbf{V_1}+\rho_2\mathbf{V_2})}{\rho_1+\rho_2} \pm i\sqrt{\rho_1\rho_2\left([\mathbf{k}\cdot(\mathbf{V_1}-\mathbf{V_2})]^2 - \frac{(\mathbf{k}\cdot\mathbf{B_1})^2+(\mathbf{k}\cdot\mathbf{B_2})^2}{4\pi\rho_{12}}\right)} \tag{2}$$

This equation is extremely useful for determining stability. It can be seen from Equation 1 that if ω

becomes imaginary, the solution will grow. Thus, if the velocity shear, magnetic fields, and densities are known, the stability of a given wavenumber (or equivalently *spatial scale*) can be determined. An example of this type of analysis can be seen in Figure 3.



(a) Im(ω) vs k for Stratified, unmagnetized flow

(b) ω vs Mach Number for magnetized, relativistic flow

Fig. 3.—: An example of linear stability analysis. ω is plotted vs some parameter to discover instabilities. Anywhere the imaginary part is positive, the solution will grow.

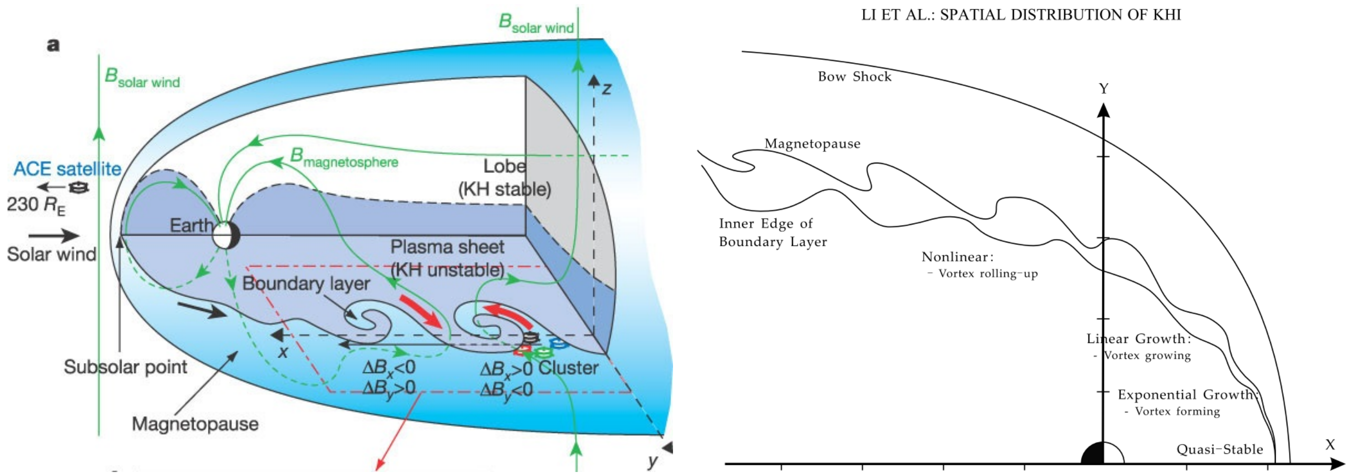
3. KH in our Magnetosphere

3.1. Where does it occur?

In Earth's Magnetosphere, there is a layer called the low-latitude boundary layer (LBLL). See Figure 4. It is made of tailward moving plasma, which has properties that are intermediate between the properties of the magnetosheath and the magnetosphere (Sonnerup 1980), and is about 800 km thick (Berchem & Russell 1982). It has been discovered that, when the Solar Wind IMF is directed Northward, the KH instability can occur in the LBLL along the MP boundary. (Miura 1985) Simulations by Li *et al.* (2012) suggest that the inner and outer edges of the LLBL are KH unstable at least between Solar wind velocities of 400 and 600 km/s, with instability beginning at about 30 degrees longitude from the subsolar point. Nonlinear growth begins at about 90 degrees on either side, and continues into the magnetotail.

3.2. Why does it matter?

The magnetopause (MP) is a barrier that surrounds the Earth. Charged particles from the solar wind are deflected by the MP, denied entry into the greater magnetosphere. One of the physical processes that can allow plasma across the magnetosphere is magnetic reconnection, where changing magnetic topology allows particles to transfer to different field lines. (Hasegawa 2012) But the KH instability provides another mechanism for the transport of mass, energy, and momentum into the magnetosphere. Fujimoto & Terasawa (1994) performed hybrid simulations of MP KHI. They showed that ion mixing across the velocity shear layer can be enhanced by the nonlinear growth of the KHI. Atkinson & Watanabe (1966) suggested that the KHI could generate ultra low frequency waves in the magnetosphere, and Elkington (2006) concluded that these waves might accelerate electrons in the outer radiation belt. Miura (1984) extensively studied momentum transport due to the KHI, and it is thought that it could drive large scale convection within the magnetosphere, especially at the MP, though the significance of the KHI for momentum transport is still unclear.



(a) LBLL KH Instability (b) Schematic of the LBLL

Fig. 4.—: The LBLL, a layer of the magnetosphere in which the KH instability occurs. It it localized to a small band in z. The solar wind is flowing in from the right in these images.

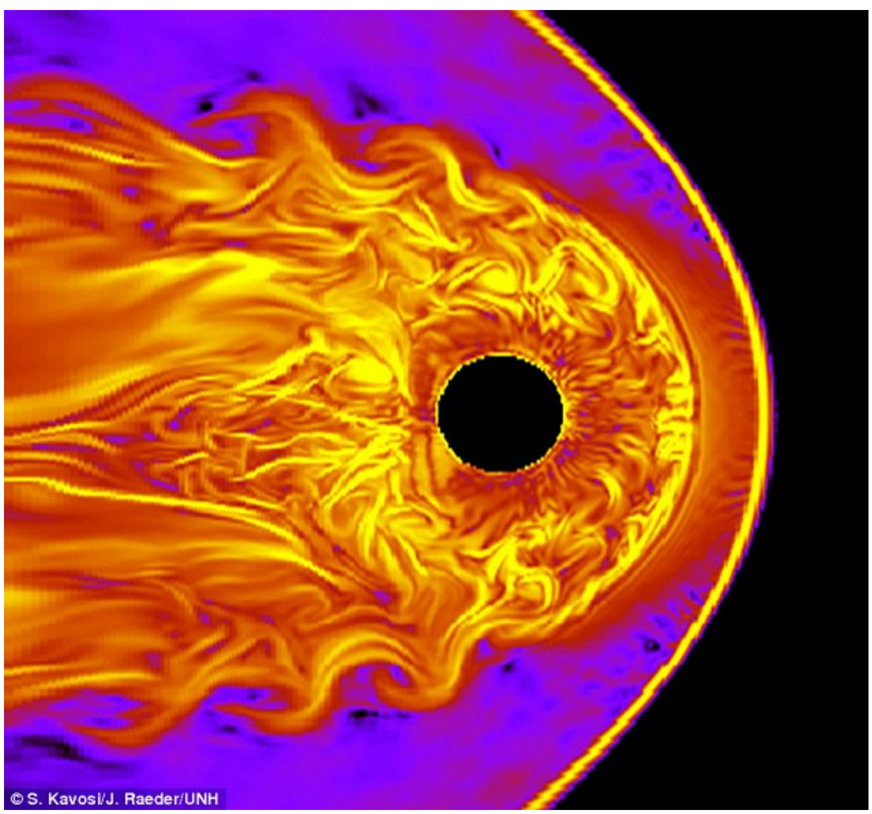


Fig. 5.—: The Kelvin-Helmholtz Instability growing on the Earth's Magnetopause in the LBLL. (Kavosi & Raeder 2015)

4. KH Elsewhere

4.1. Saturn and Jupiter

This instability also occurs in the magnetospheres of the Jovian Planets. At Jupiter, most of the dawn flank of the MP is KH unstable, regardless of the symmetry of the MP. The dusk flank, tailward of the planet, is also unstable, but only when the MP is highly oblate. (Desroche *et al.* 2012) In contrast, at Saturn most of the dawn and dusk equatorial region of the MP is unstable, due to the presence of the dense MSP plasma sheet and weak magnetic fields on either side of the MP. The stability of the system at Saturn is much more dependent on the angle of the incident Solar wind IMF than it is at Jupiter. (Desroche *et al.* 2013) At Saturn, the MP boundary may be most sensitive to the KHI in the subsolar region, and the vortices may then be transported to the dusk side by corotational flow. (Delamere *et al.* 2013)

4.2. Mercury

Though Mercury has a weak dipole field, the relatively strong solar wind field leads to highly dynamic magnetospheric processes. In April 2011, the MESSENGER spacecraft visited Mercury . While there, it frequently observed KH waves in the subsolar magnetosphere, with instability growth rates much larger than those at earth. These waves involve plasma transfer into the magnetosphere, and could be a source for the thick LLBL observed at the planet. A clear dawn-dusk asymmetry was observed, with most KH activity observed between noon and dusk.(Sundberg *et al.* 2012) Further MESSENGER observations indicate that simple MHD KH waves are occurring on the dayside, which become smaller, kinetic scale KH waves as they are swept to the night side. (Gershman *et al.* 2015)

5. Summary and Discussion

The Kelvin-Helmholtz Instability occurs in many places in our solar system. Both magnetized and non-magnetized bodies are candidates for hosting this instability, which usually occurs at the magnetopause in a thin equatorial region of the magnetosphere called the Low Latitude Boundary Layer. This instability is important because it allows mass, energy, and momentum to efficiently cross magnetic boundaries. That being said, the details of the way that these transport processes work are still unknown, and an active area of research. Until we fully understand the role that this instability plays in each of these locations, we will be unable to fully account for all the dynamics of these systems.

Figures†

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Brekenridge: https://img.washingtonpost.com/blogs/capital-weather-gang/files/2015/11/Breckenridge.jpg&w=480
LLBL purple simulation: http://www.dailymail.co.uk/sciencetech/article-3155432
        /The-peculiar-pattern-universe-Nasa-shots-reveal-striking-surfer-waves-surrounding-Earth-Saturn.html
Solar KH: https://www.slideshare.net/niclabrosse/solar-prominence-science-with-alma?from_action=save
Ocean Currents, Saturn Atmosphere: Wikimedia Commons
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REFERENCES

Atkinson, Gerald, & Watanabe, Tomiya. 1966. Surface Waves on the Magnetospheric Boundary as a possible Origin of Long Period Geomagnetic Micropulsations. *EARTH AND PLANETARY SCIENCE LETTERS*, **1**, 89–91.

Berchem, Jean, & Russell, C. T. 1982. The thickness of the magnetopause current layer: ISEE 1 and 2 observations. *Journal of Geophysical Research*, **87**(A4), 2108.

Delamere, P. A., Wilson, R. J., Eriksson, S., & Bagenal, F. 2013. Magnetic signatures of Kelvin-Helmholtz vortices on Saturn's magnetopause: Global survey. *Journal of Geophysical Research: Space Physics*, **118**(1), 393–404.

Desroche, M., Bagenal, F., Delamere, P. A., & Erkaev, N. 2012. Conditions at the expanded Jovian magnetopause and implications for the solar wind interaction. *Journal of Geophysical Research: Space Physics*, **117**(A7), n/a–n/a.

Desroche, M., Bagenal, F., Delamere, P. A., & Erkaev, N. 2013. Conditions at the magnetopause of Saturn and implications for the solar wind interaction. *Journal of Geophysical Research: Space Physics*, **118**(6), 3087–3095.

Elkington, Scot R. 2006. A Review of ULF Interactions with Radiation Belt Electrons. *THE RADIATION BELTS*.

Fujimoto, M., & Terasawa, T. 1994. Anomalous ion mixing within an MHD scale Kelvin-Helmholtz vortex. *Journal of Geophysical Research*, **99**(A5), 8601.

Gershman, Daniel J., Raines, Jim M., Slavin, James A., Zurbuchen, Thomas H., Sundberg, Torbjörn, Boardsen, Scott A., Anderson, Brian J., Korth, Haje, & Solomon, Sean C. 2015. MESSENGER observations of multiscale Kelvin-Helmholtz vortices at Mercury. *Journal of Geophysical Research: Space Physics*, **120**(6), 4354–4368.

Hasegawa, Hiroshi. 2012. Structure and Dynamics of the Magnetopause and Its Boundary Layers. *Monographs on Environment, Earth and Planets*, **1**(2), 71–119.

Johnson, Jay R., Wing, Simon, & Delamere, Peter A. 2014. Kelvin Helmholtz Instability in Planetary Magnetospheres. *Space Science Reviews*, **184**(1-4), 1–31.

Kavosi, S., & Raeder, J. 2015. Kelvin-Helmholtz Instability under southward IMF: THEMIS observations and OpenGGCM simulations. *American Geophysical Union, Fall Meeting 2015, abstract #SM21C-01.*

Li, W. Y., Guo, X. C., & Wang, C. 2012. Spatial distribution of Kelvin-Helmholtz instability at low-latitude boundary layer under different solar wind speed conditions. *Journal of Geophysical Research: Space Physics*, **117**(A8), n/a–n/a.

Mishin, V. V., & Tomozov, V. M. 2016. KelvinHelmholtz Instability in the Solar Atmosphere, Solar Wind and Geomagnetosphere. *Solar Physics*, **291**(11), 3165–3184.

Miura, Akira. 1984. Anomalous transport by magnetohydrodynamic Kelvin-Helmholtz instabilities in the solar wind-magnetosphere interaction. *Journal of Geophysical Research*, **89**(A2), 801.

Miura, Akira. 1985. Kelvin-Helmholtz instability at the magnetospheric boundary. *Geophysical Research Letters*, **12**(10), 635–638.

Sonnerup, Bengt U. Ö. 1980. Transport Mechanisms at the Magnetopause. *Pages 77–100 of: Dynamics of the Magnetosphere.*

Sundberg, Torbjörn, Boardsen, Scott A., Slavin, James A., Anderson, Brian J., Korth, Haje, Zurbuchen, Thomas H., Raines, Jim M., & Solomon, Sean C. 2012. MESSENGER orbital observations of large-amplitude Kelvin-Helmholtz waves at Mercury's magnetopause. *Journal of Geophysical Research: Space Physics*, **117**(A4), n/a–n/a.

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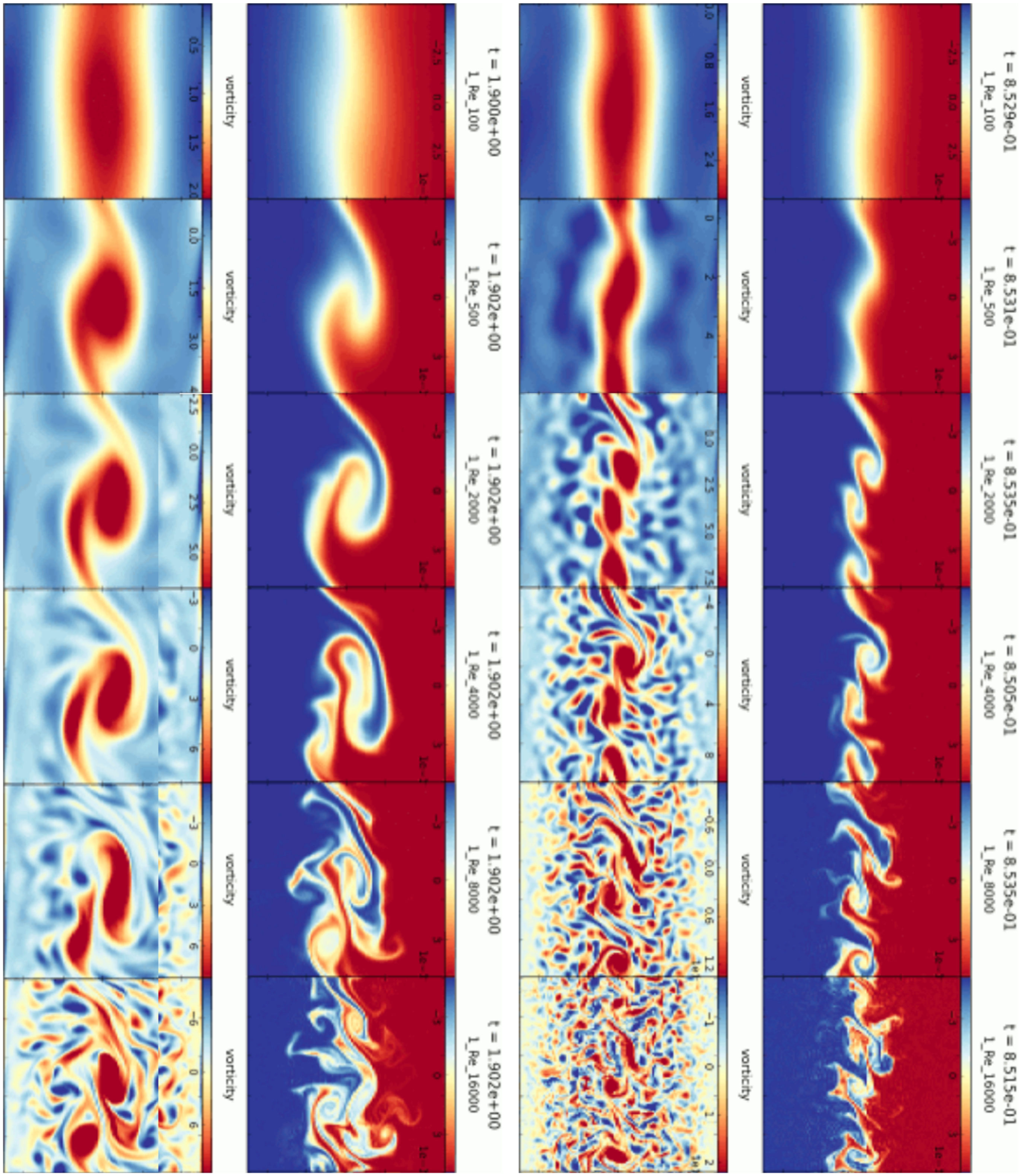

Fig. 6.—: This simulation, performed in Dedalus by Chris Gilbert in Fall 2015, generates the Kelvin-Helmholtz Instability in fluids with varying Reynold's Number. There is a low Re cutoff, below which diffusion is too strong for the instability to occur. There is also a high Re cutoff, above which the fluid is too cohesive, and breaks up into turbulence too quickly for the instability to grow. The simulation is shown at two different time steps.